Mr. Wright's Math Extravaganza

Physical Sciences
(Chemistry, Physics, Physical Science)
Gravity
Unit 05 Kepler's Laws and Gravity

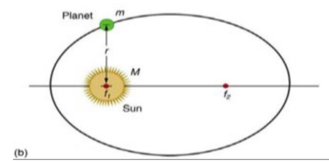
Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on satellite crash lab

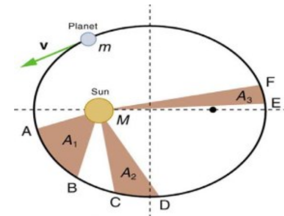
Score	I Can Statements
4.0	☐ I can investigate the outcome of a collision between two satellites.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	☐ I can use Newton's law of gravitation to describe the gravitational forces between objects. ☐ I can predict the motion of orbiting objects in the solar system.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	☐ I can use Newton's law of gravitation to explain the proportional relationships between the force of gravity, mass, and the distance between two objects. ☐ I can explain the forces between the earth and a person standing on the surface of the Earth. ☐ I can substitute values for variables, constants, and applicable equations as necessary. ☐ I can use Newton's second law of motion and law of gravitation as well as the equation for acceleration to find an object's velocity in a circular orbit. ☐ I can use $2\pi r$ and an object's velocity, to determine the period and frequency of an object in a circular orbit. ☐ I can explain each of Kepler's laws of planetary motion in relation to knowledge of orbiting objects. ☐ I can diagram an object in space's elliptical orbit.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Physics 05-01 Kepler's Laws of Planetary Motion **Name: ______________**

- After studying motion of planets, ______________ came up with his laws of planetary motion
- ______________ then proved them all using his Universal Law of Gravitation
- Assumptions:
  - A ______________ mass, m, orbits much ______________ mass, M, so we can use M as an approximate inertia reference frame
  - The system is ______________

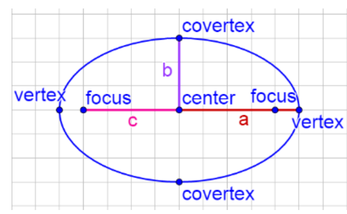
Kepler's Laws





1. The ______________ of each planet about the Sun is an ______________ with the sun at one ______________.
   a. Closest point to the sun __________________
   b. Farthest point to the sun ____________________
2. Each ______________ moves so that an ______________ line drawn from the ______________ to the ______________ sweeps out equal ______________ in equal ______________.
3. The ______________ of the ______________ of the ______________ of any two planets about the sun is equal to their ______________ of the ______________ of their average ______________ from the sun.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$



- Parts of an ellipse
  - a = __________________axis (distance from ___________to __________) $a = \frac{r_a + r_p}{2}$
  - b = __________________axis (distance from ___________to __________) $b = \sqrt{r_a r_p}$
  - c = ______________length (distance from ____________to __________)$c = r_a - a$
  - Area of ellipse: $A = \pi ab$

Eccentricity

- Measure of how ___________________an ellipse is $e = \frac{c}{a}$
- e = 0 _____________; e = 1 _____________

The perihelion of the moon from earth is 358000 km. Its aphelion is 399000 km. What is the moon's orbit's semimajor axis, semiminor axis, focal length, and eccentricity?

If it takes 27.3 days for the moon to orbit the earth, how much area does a line from the earth to the moon sweep out every day?

Physics 05-01 Kepler's Laws of Planetary Motion *Name: _______________*

The moon's average radius of orbit is 384,399 km and takes 27.322 days to orbit the earth. The International Space Station's average radius of orbit is 417.5 km above the earth. What is the period of the ISS's orbit?

Practice Work

1. Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.
2. Are Kepler's laws purely descriptive, or do they contain causal information?
3. Comets have very elongated elliptical orbits with the sun at one focus. Using Kepler's Law, explain why a comet travels much faster near the sun than it does at the other end of the orbit. (HSP 7.2)
4. Explain how the masses of a satellite and its parent body must compare in order to apply Kepler's laws of planetary motion. (HSP 7.27)
5. The orbit of Halley's Comet has an eccentricity of 0.967 and stretches to the edge of the solar system.
(a) Describe the shape of the comet's orbit.
(b) Compare the distance traveled per day when it is near the sun to the distance traveled per day when it is at the edge of the solar system.
(c) Describe variations in the comet's speed as it completes an orbit. Explain the variations in terms of Kepler's second law of planetary motion. (HSP 7.35)
6. A moon orbits a planet in an elliptical orbit. The foci of the ellipse are 50,000 km apart. The closest approach of the moon to the planet is 400,000 km. What is the length of the major axis of the orbit? (HSP 7.12) **850,000 km**
7. The focal point of the elliptical orbit of a moon is 50000 km from the center of the orbit. If the eccentricity of the orbit is 0.25, what is the length of the semi-major axis? (HSP 7.21) **200,000 km**
8. An artificial satellite orbits the Earth at a distance of 1.45×10^4 km from Earth's center. The moon orbits the Earth at a distance of 3.84×10^5 km once every 27.3 days. How long does it take the satellite to orbit the Earth? (HSP 7.22) **0.200 days**
9. Earth is 1.496×10^8 km from the sun, and Venus is 1.08×10^8 km from the sun. One day on Venus is 243 Earth days long. What best represents the number of Venusian days in a Venusian year? (HSP 7.23) **0.92 days**
10. Mars has two moons, Deimos and Phobos. The orbit of Deimos has a period of 1.26 days and an average radius of 2.35×10^3 km. The average radius of the orbit of Phobos is 9.374×10^3 km. According to Kepler's third law of planetary motion, what is the period of Phobos? (HSP 7.30) **10.0 days**

Physics 05-02 Weight and Gravity **Name: ____________________**

Newton's Law of Universal Gravitation

Every __________ in the universe exerts a __________ on every other particle

$$F_G = \frac{GmM}{r^2}$$

where:

$G = 6.673 \times 10^{-11} \frac{Nm^2}{kg^2}$

m and M=__________of the objects

r = __________ between the __________ of the objects

What is the gravitational attraction between a 75-kg boy (165 lbs) and the 50-kg girl (110 lbs) seated 1 m away in the next desk?

Finding Acceleration Due to Gravity

Since weight is the __________ of __________

$$W = mg = \frac{GmM}{r^2}$$

$$g = \frac{GM}{r^2}$$

Find the acceleration due to gravity at the altitude of the ISS, 417.5 km above the earth.

Practice Work

1. How are weight and mass related? How are they different?
2. If the distance between two objects triples, what happens to the magnitude of the gravitation force between them?
3. When calculating the acceleration due to gravity, which mass do you use?

4. A bowling ball (mass = 7.2 kg, radius = 0.11 m) and a billiard ball (mass = 0.38 kg, radius = 0.028 m) may each be treated as uniform spheres. What is the magnitude of the maximum gravitational force that each can exert on the other? (Cutnell 4.18) $\mathbf{9.6 \times 10^{-9}\ N}$

5. On earth, two parts of a space probe weight 11000 N and 3400 N. These parts are separated by a center-to-center distance of 12 m and may be treated as uniform spherical objects. Find the magnitude of the gravitational force that each part exerts on the other out in space, far from any other objects. (Cutnell 4.19) $\mathbf{1.8 \times 10^{-7}\ N}$

6. What is the gravitational force between the earth, $m = 5.98 \times 10^{24}\ kg$, and the sun, $m = 1.99 \times 10^{30}\ kg$, if they are separated by $1.48 \times 10^{8}\ km$? (RW) $\mathbf{3.62 \times 10^{22}}\ \boldsymbol{N}$

7. If Venus orbits the sun at 1.08×10^{8} km and experiences a gravitational force of 5.54×10^{22} N, what is its mass? (RW) $\mathbf{4.87 \times 10^{24}}\ \boldsymbol{kg}$

8. What is the acceleration due to gravity on the surface of the Moon? (OpenStax 6.35a) **1.62 m/s²**

9. What is the acceleration due to gravity on the surface of Mars? The mass of Mars is 6.418×10^{23} kg and its radius is 3.38×10^{6} m. (OpenStax 6.35b) **3.75 m/s²**

10. (a) Calculate the acceleration due to gravity on the surface of the Sun. (b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.) (OpenStax 6.36) **274 m/s², 28 times**

11. What is the acceleration due to gravity as an altitude of 2.0×10^{6} m above the earth's surface? (RW) **5.68 m/s²**

Physics 05-03 Satellites **Names: ___________________**

- Any object _____________ another object only under the influence of _____________
- One way to find the speed of a satellite in a ___________________ orbit
- $v = \frac{2\pi r}{T}$
  - Where r = orbital radius, T = period of orbit
- Gravity provides the _____________ force

There is only _____________ speed that a satellite can have if the satellite is to remain in an orbit with a _____________ radius.

$$v = \sqrt{\frac{GM}{r}}$$

- r is measured from _____________ of the Earth
- As r _____________, v_____________
- _____________ of the satellite is not in equation

Calculate the speed of a satellite 500 km above the earth's surface.

Find the mass of a black hole where the matter orbiting it at r = 2.0×10^{20} m move at speed of 7,520,000 m/s.

Kelpler's Third Law (for circular orbits)

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Or

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Use the data of Mars to find the mass of the sun assuming a circular orbit. ($r = 2.279 \times 10^8$ km, $T = 1.881$ yr)

Physics 05-03 Satellites *Names: ___________________*

Since satellites are moving only under the influence of _____________, and the acceleration points towards _____________, satellites are in _____________.

Practice Work

1. Derive the formula for the speed of a satellite in a circular orbit around a planet of mass M.
2. What is the speed of a satellite 1.0×10^3 km above an unknown planet of radius 6.38×10^6 m if it takes 1.75 hours to complete one orbit? (RW) **7360 m/s**
3. A satellite is in a circular orbit around an unknown planet. The satellite has a speed of 1.70×10^4 m/s, and the radius of the orbit is 5.25×10^6 m. A second satellite also has a circular orbit around this same planet. The orbit of this second satellite has a radius of 8.60×10^6 m. What is the speed of the second satellite? (Cutnell 5.27) $\mathbf{1.33 \times 10^4\ m/s}$
4. A satellite is placed in orbit 6.00×10^5 m above the surface of Jupiter. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.14×10^7 m. Find the orbital speed of the satellite. (Cutnell 5.29) $\mathbf{4.20 \times 10^4\ m/s}$
5. The moon orbits the earth at a distance of 3.85×10^8 m. Assume that this distance is between the centers of the earth and the moon and that the mass of the earth is 5.98×10^{24} kg. Find the period for the moon's motion around the earth. Express the answer in days and compare it to the length of a month. (Cutnell 5.30) **27.5 days**
6. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit. (OpenStax 6.43) $\mathbf{4.23 \times 10^4\ km}$
7. Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass. (OpenStax 6.44) $\mathbf{1.99 \times 10^{30}\ kg}$
8. Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass. (OpenStax 6.45) $\mathbf{1.90 \times 10^{27}\ kg}$
9. Astronomical observations of our Milky Way galaxy indicate that it has a mass of about 8.0×10^{11} solar masses. A star orbiting on the galaxy's periphery is about 6.0×10^4 light years from its center. (a) What should the orbital period of that star be? (b) If its period is 6.0×10^7 years instead, what is the mass of the galaxy? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies. (OpenStax 6.47) $\mathbf{3 \times 10^8}$ **years,** $\mathbf{2 \times 10^{13}}$ **solar masses**

Physics Unit 5: Kepler's Laws and Gravity Review

1. Know about Kepler's Laws of Planetary Motion, eccentricity, perihelion, aphelion, weight, mass, gravitational force, acceleration due to gravity, centripetal force, speed of a satellite in orbit
2. When does a satellite travel the fastest?
3. Why do astronauts seem to float?
4. A moon's average radius of orbit is 2×10^4 km and takes 20 days to orbit the earth. A communications satellite takes 3 days to orbit the earth. What is the orbital radius of the satellite?
5. The perihelion of an asteroid is 3×10^8 km, and its aphelion is 5×10^8 km. What is its orbit's eccentricity?
6. A satellite has an orbital with a perihelion of 3200 km and an aphelion of 3400 km both measured from the center of the earth. If it takes 65 minutes to orbit the earth, how much area does a line from the satellite to the earth sweep out every minute?
7. If the distance between two objects is divided by 4, what happens to the gravitation force between them?
8. Find the gravitational force of attraction between a 30-kg girl and a 40-kg boy sitting 0.5 meters apart.
9. A planet with mass 8×10^{20} kg orbits a star with mass 9×10^{30} kg. If the gravitational force between them is 6×10^{18} N, what is the planet's orbital radius?
10. What is the acceleration due to gravity at an altitude of 200 km above the earth's surface? **Note:** the radius of the earth is 6.36×10^6 m; the mass of the earth is 5.98×10^{24} kg.
11. What is the mass of a moon where the acceleration due to gravity is 7.0 m/s^2 at its surface? **Note:** the radius of the moon is 2×10^4 m.
12. Calculate the speed of a 120 kg satellite in orbit 425 m above a moon with mass 2×10^{22} kg and radius 8×10^5 m.
13. A starship is orbiting a planet at 1200 m/s. Calculate the mass of the planet if the radius of the starship's orbit is 8×10^5 m.
14. An asteroid is orbiting a star at 8×10^{10} m. If the mass of the star is approximately 2×10^{29} kg, what is the period of the asteroid's orbit?
15. A planet takes 615 days to orbit its star at a distance of 2×10^{12} m. What is the mass of the star?

Physics Unit 5: Kepler's Laws and Gravity Review

Answers

2. When it is closest to the main body such as the sun or planet
3. They are in freefall with their surroundings
4. $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

   $\frac{(3\ days)^2}{(20\ days)^2} = \frac{r^3}{(2\times10^4\ km)^3}$

   $r^3(400\ days^2) = 7.2\times10^{13}\ days^2 km^3$

   $r^3 = 1.8\times10^{11}\ km^3$

   $r = \sqrt[3]{1.8\times10^{11}} \approx \mathbf{5650\ km}$
5. $e = \frac{c}{a}$

   $a = \frac{r_a+r_p}{2} = \frac{3\times10^8\ km+5\times10^8\ km}{2} = 4\times10^8\ km$

   $c = r_a - a = 5\times10^8\ km - 4\times10^8\ km =$
   $1\times10^8\ km$

   $e = \frac{c}{a} = \frac{1\times10^8\ km}{4\times10^8\ km} = \mathbf{0.25}$
6. $a = \frac{r_a+r_p}{2} = \frac{3200\ km+3400\ km}{2} = 3300\ km$

   $b = \sqrt{r_a r_b} = \sqrt{(3200\ km)(3400\ km)} = 3298\ km$

   $A = \pi ab$

   $A = \pi(3300\ km)(3298\ km) = 3.42\times10^7\ km^2$

   $\frac{A}{min} = \frac{3.42\times10^7\ km^2}{65\ \text{min}} = \mathbf{5.26\times10^5\ km^2/min}$
7. $F = \frac{GmM}{r^2}$ so $F \propto \frac{1}{r^2}$.

   $F \propto \frac{1}{\left(\frac{1}{4}\right)^2}$

   $F \propto 16$

   ***F* is multiplied by 16.**
8. $F = \frac{GMm}{r^2}$

   $F = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(30\ kg)(40\ kg)}{(0.5\ m)^2}$

   $F = \mathbf{3.2\times10^{-7}\ N}$
9. $F = \frac{GMm}{r^2}$

   $6\times10^{18}\ N = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(8\times10^{20}\ kg)(9\times10^{30}\ kg)}{r^2}$

   $r^2 = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(8\times10^{20}\ kg)(9\times10^{30}\ kg)}{6\times10^{18}\ N}$

   $r = \sqrt{8.004\times10^{22}\ m^2} = \mathbf{2.83\times10^{11}\ m}$
10. $g = \frac{GM}{r^2}$

    $g = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(5.98\times10^{24}\ kg)}{(6.36\times10^6\ m+200\times10^3\ m)^2}$

    $g = \mathbf{9.27\ m/s^2}$
11. $g = \frac{GM}{r^2}$

    $7.0\frac{m}{s^2} = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)M}{(2\times10^4\ m)^2}$

    $2.8\times10^9\frac{m^3}{s^2} = \left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)M$

    $\mathbf{4.20\times10^{19}\ kg} = M$
12. $v = \sqrt{\frac{GM}{r}}$

    $v = \sqrt{\frac{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(2\times10^{22}\ kg)}{425\ m+8\times10^5\ m}} = \mathbf{1290\ m/s}$
13. $v = \sqrt{\frac{GM}{r}}$

    $1200\frac{m}{s} = \sqrt{\frac{\left(6.67\times\frac{10^{-11}Nm^2}{kg^2}\right)M}{8\times10^5\ m}}$

    $1.44\times10^6\frac{m^2}{s^2} = \left(8.34\times10^{-17}\frac{Nm}{kg^2}\right)M$

    $\mathbf{1.73\times10^{22}\ kg} = M$
14. $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

    $\frac{T^2}{(8\times10^{10}\ m)^3} = \frac{4\pi^2}{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(2\times10^{29}\ kg)}$

    $T^2 = \frac{4\pi^2(8\times10^{10}\ m)^3}{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)(2\times10^{29}\ kg)}$

    $T = \sqrt{1.52\times10^{15}\ s^2} = \mathbf{3.89\times10^7\ s}$
15. $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

    $\frac{615\ days}{1}\left(\frac{24\ hr}{1\ day}\right)\left(\frac{3600\ s}{1\ hr}\right) = 5.31\times10^7\ s$

    $\frac{(5.31\times10^7\ s)^2}{(2\times10^{12}\ m)^3} = \frac{4\pi^2}{\left(6.67\times10^{-11}\frac{Nm^2}{kg^2}\right)M}$

    $\left(1.88\times10^5\frac{m^3}{kg}\right)M = 3.16\times10^{38}\ m^3$

    $M = \mathbf{1.68\times10^{33}\ kg}$